UNIVERSITY OF LEICESTER

School of Engineering

Course Introduction

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EG1101 – Mechanical Engineering – Mechanics of Materials

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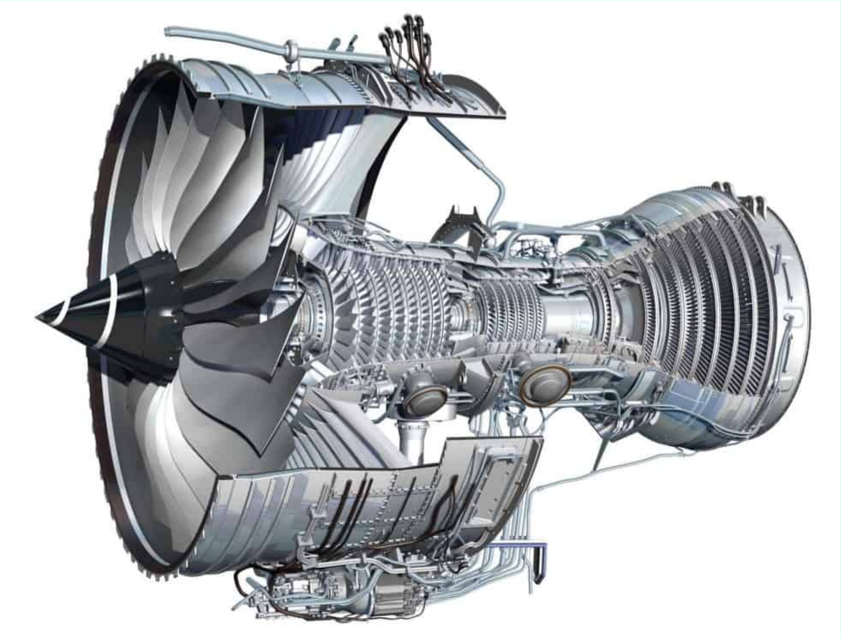
EG1101 Mechanical Engineering - Introduction

- 30 credits:
  - Thermodynamics
  - Fluid mechanics
  - Mechanics of Materials

- Mechanics of Materials:
  - 28 Lectures:

    2 Pre-recorded lectures/week

    1 Live lecture/week



Rolls Royce Trent 1000 engine

- Solid mechanics assessment:
  - Exam - May 2022

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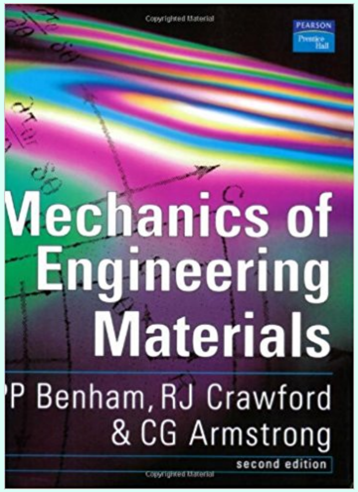
Mechanics of Materials

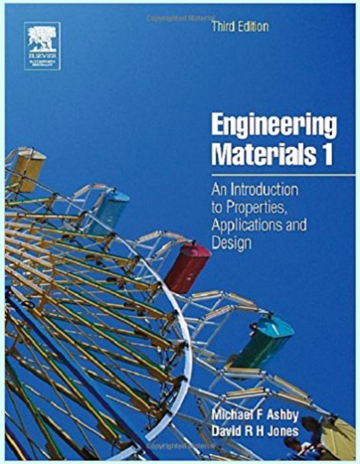
Reference book (recommended reading):

- **Mechanics of Engineering Materials**
  P.P. Benham, R.J. Crawford and C.G. Armstrong
  Longman Scientific & Technical



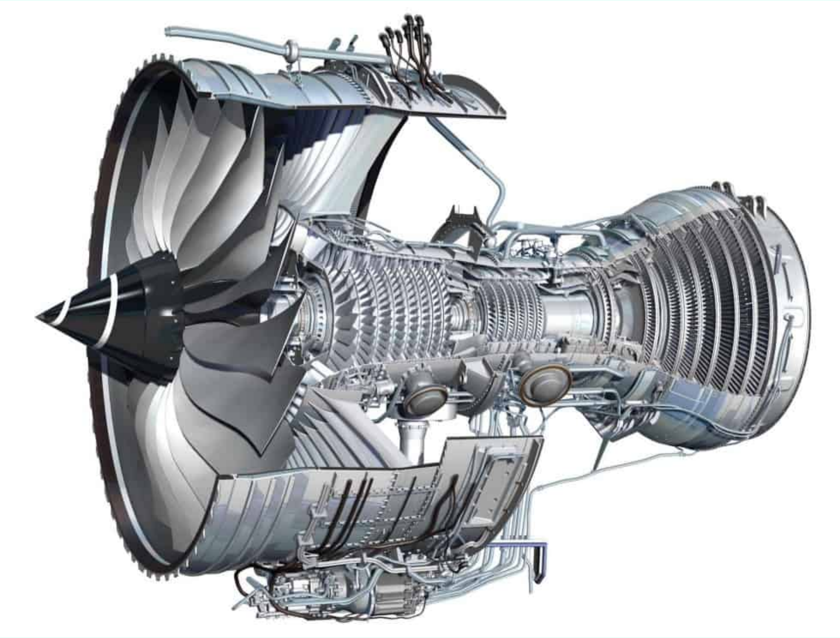
Background reading:

- **Engineering Materials 1**
  M.F. Ashby and D.R.H. Jones
  Butterworth Heinemann



Topics covered in EG1101 Mechanics of Materials

- **Topic 1** Stress and strain and their relation.
- **Topic 2** Static Equilibrium and Free Body Diagrams
- **Topic 3** Beam Bending theory
- **Topic 4** Stress in thin walled pressure vessels
- **Topic 5** Shear stress and torsion of cylinders



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Stress, strain and their relation

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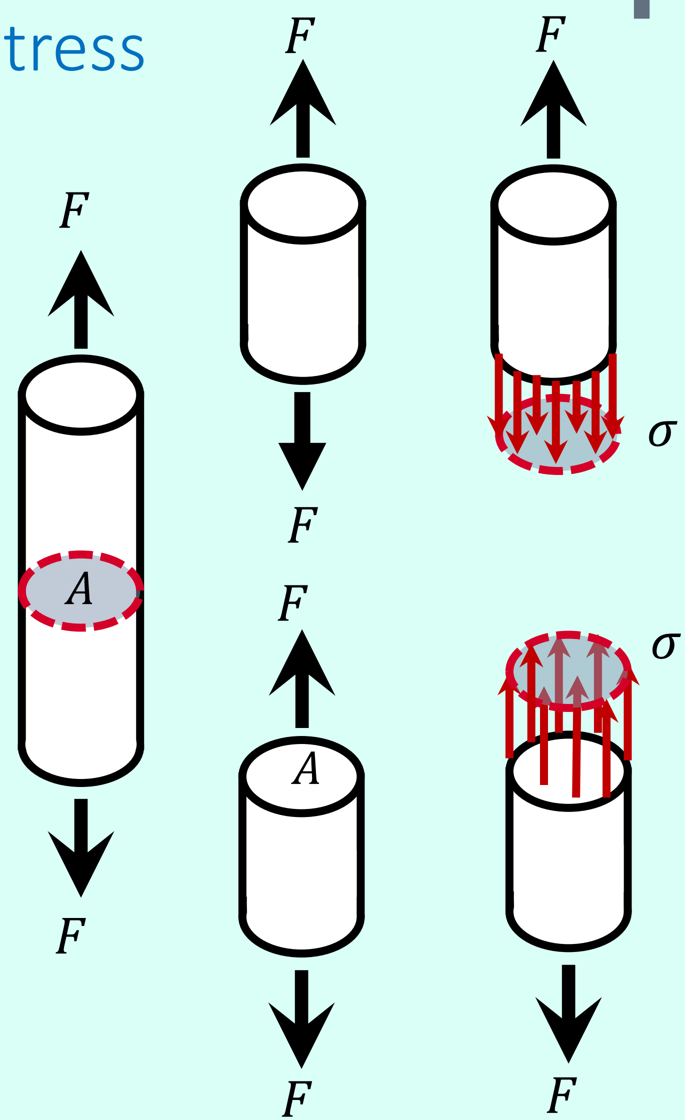
Mechanical Behaviour of Materials – Stress

Nominal stress – the stress due to a normal internal loading/force (F) along the main axis of the deformable body 'unit area (A)'

$$\text{Nominal stress}, \sigma_n = \frac{\text{Force}}{\text{Initial area}} = \frac{F}{A}$$

Dimensionally stress is ${}^{F}/_{L^2} = M.L^{-1}.T^{-2}$



Mechanical Behaviour of Materials – Stress units

In SI units (N, m, s)

Stress is N/m^2 – Pascal (Pa)

Alternative unit scheme (N, mm, s)

Giving Stress in $N/mm^2 - MPa$

(a more convenient unit for stress)

Stress: By Definition

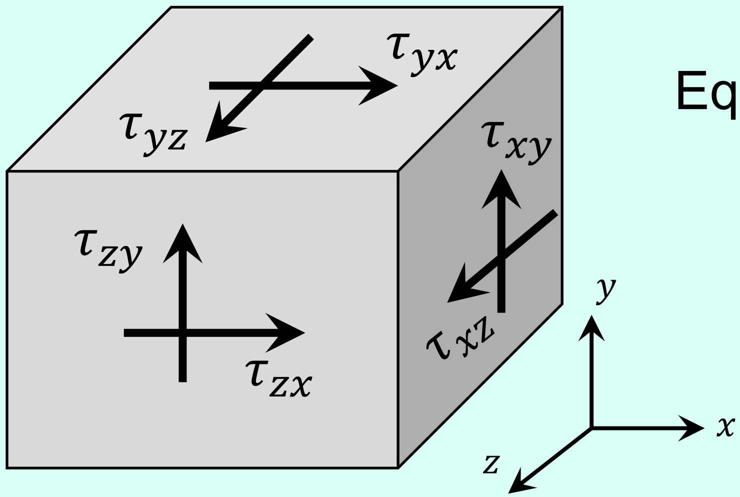
Positive (+) stresses show material is in **Tension** (Tensile stress)

Negative (-) stresses show material is in **Compression** (Compressive stress)

Mechanical Behaviour of Materials – Shear stress

Shear stress – the stress due to an internal loading/force (F) that is parallel to the cross-sectional plane of interest (A)

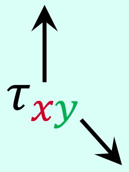
Shear stress, $\tau = {}^{F}/_{A}$



Equilibrium condition:

$$\tau_{xy} = \tau_{yx}$$
$$\tau_{xz} = \tau_{zx}$$
$$\tau_{zy} = \tau_{yz}$$

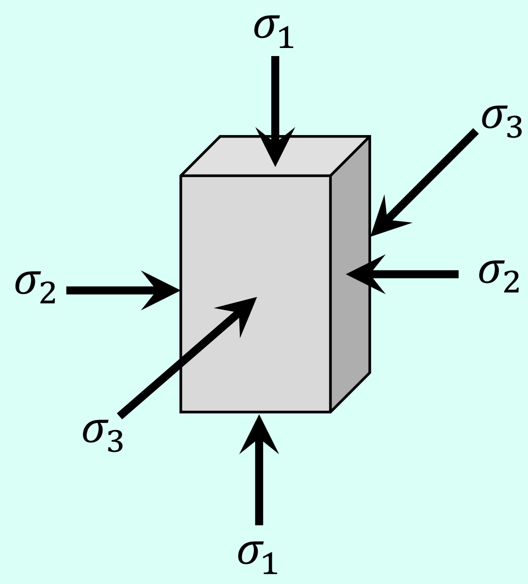
The direction of the normal to the plane on which the stress is acting, i.e., the axis that's perpendicular to the plan.



The direction of the shear stress component.

Mechanical Behaviour of Materials – Hydrostatic stress

Hydrostatic Stress – is a component of stress which contains uniaxial stresses, but not shear stresses. It can be represented by the stress set up in a body immersed at a great depth in a fluid.



$$\text{Hydrostatic stress, } \sigma_H = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

Mechanical Behaviour of Materials – Strain

Nominal strain – Defined as change of length per unit of length

$$\text{Nominal Strain}, \epsilon_n = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L}$$

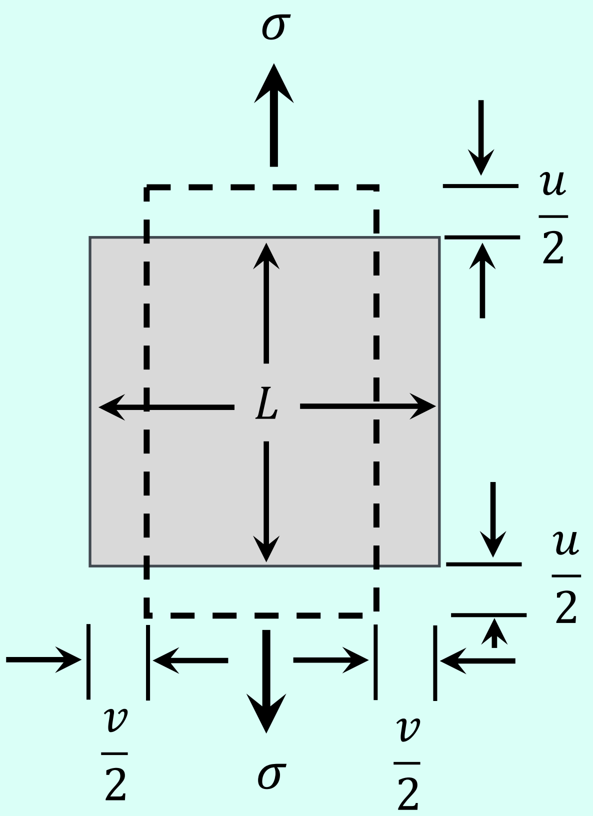
Nominal tensile strain:

$$\epsilon_y = {}^{u}/_{L}$$

Nominal lateral strain:

$$\epsilon_x = {}^{v}/_{L}$$

The relationship between them is the Poisson's ratio, ν.



Poisson's ratio

Nominal tensile strain:

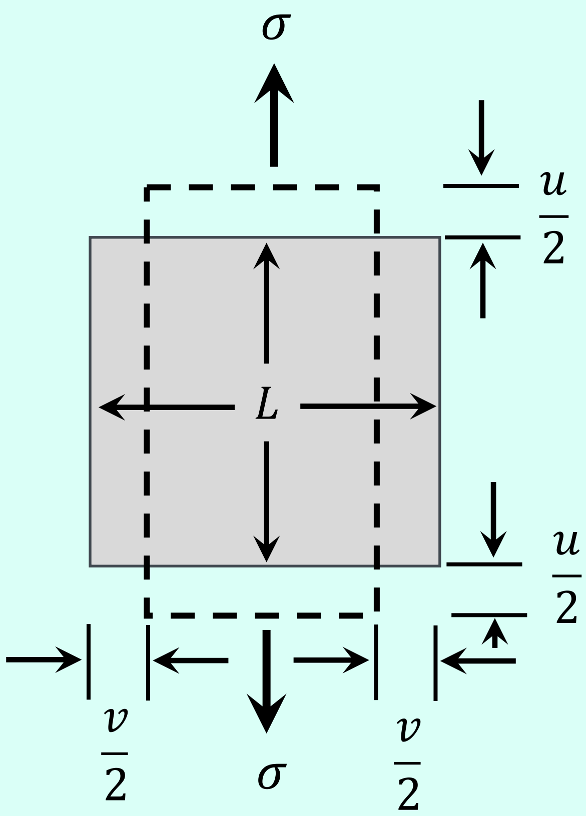
$$\epsilon_y = {}^{u}/_{L}$$

Nominal lateral strain:

$$\epsilon_x = {}^{v}/_{L}$$

$$\upsilon = -\frac{\text{lateral strain}}{\text{tensile strain}} = -\frac{\epsilon_x}{\epsilon_y}$$

$$\epsilon_x = -\nu\, \epsilon_y$$



Typical Values of Poisson's Ratio

Material	Poisson's Ratio
Cork	0
Concrete	$0.1 - 0.2$
Foam	$0.10 - 0.50$
Glass	$0.18 - 0.3$
Sand	$0.20 - 0.45$
Cast Iron	$0.21 - 0.26$
Magnesium	$0.252 - 0.289$
Titanium	$0.265 - 0.34$
Steel	$0.27 - 0.30$
Stainless Steel	$0.30 - 0.31$
Clay	$0.30 - 0.45$
Aluminium-alloy	0.32
Copper	0.33
Saturated clay	$0.40 - 0.49$
Gold	$0.42 - 0.44$
Rubber	0.4999

Poisson's Ratio

- Material Property
- Maximum Value 0.5
- Typical Value 0.3

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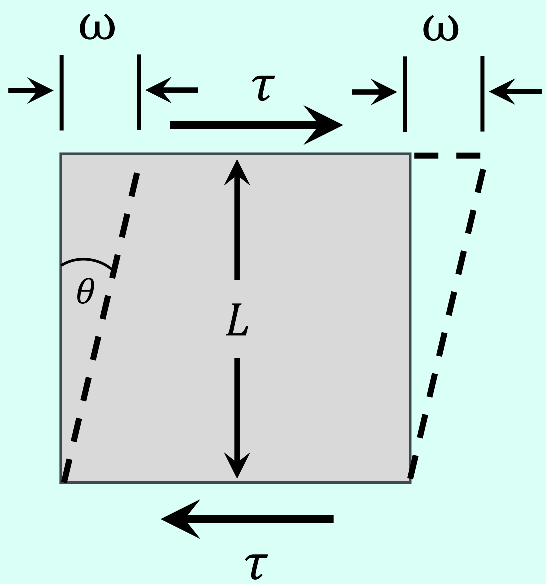
Mechanical Behaviour of Materials – Shear strain

Shear strain – is the change in angle that occurs between two-line segments that were originally perpendicular to one another.



$$\text{Shear strain}, \gamma = \frac{\omega}{L} = \tan\theta$$

ω is the angle of shear and L is the edge–length of the cube.

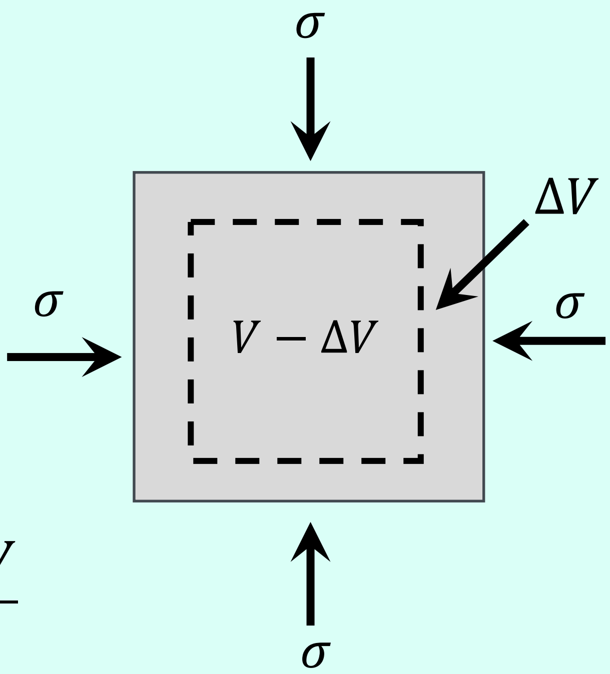
- For elastic strain (very small):

$$\gamma = \theta$$

Mechanical Behaviour of Materials – Volume strain

Hydrostatic stress produces a volume change called **volumetric strain or dilatation.**

If the volume change is "ΔV" and the cube volume is "V", we define the dilatation "Δ" by:



$$\text{Dilatation}, \Delta = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

Mechanical Behaviour of Materials – Fact about strain

- Strain has NO units “dimensionless” as it is the ratio of two lengths or of two volumes.
- Normal strains are measured in one particular direction while shear strains are measured in between two perpendicular lines.

Positive (+) strains show an increase in length/volume

Negative (-) strains show a decrease in length/volume

Mechanical Behaviour of Materials – True stress

Engineering stress and strain do no accurately represent the instantaneous stress and strain observed within a specimen during tensile loading

- **<u>True stress</u>** is calculated from the <u>instantaneous area</u> at any load:

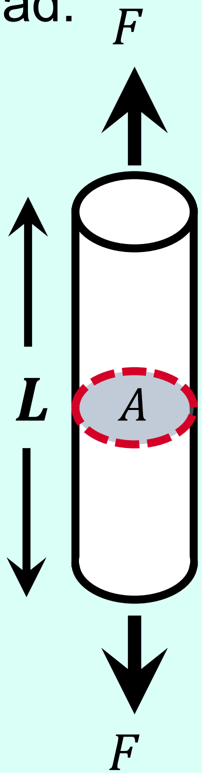
$$\text{True stress}, \sigma_t = \frac{\text{Force}}{\text{Instantaneous area}} = \frac{F}{A_i}$$

Neglecting volume change during plastic deformation:

$$\frac{A_i}{A_0} = \frac{L_0}{L_i}$$

$$\sigma_t = \frac{F}{A_i} = \frac{F L_i}{A_0 L_0} \Rightarrow \sigma_t = \frac{F}{A_0}(1 + \left(\frac{L_i}{L_0}\right))$$

$$\boxed{\sigma_t = \sigma_n(1 + \epsilon_n)}$$



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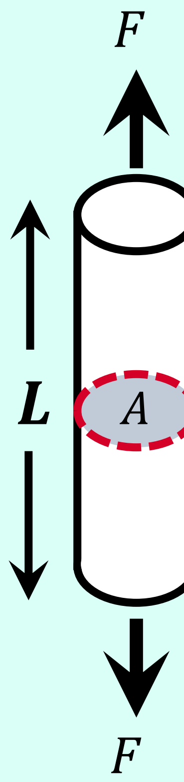
Mechanical Behaviour of Materials – True strain

Engineering stress and strain do no accurately represent the instantaneous stress and strain observed within a specimen during tensile loading

- **True tensile strain** = rate of instantaneous increase in the instantaneous gauge length.

$$\text{True strain}, \epsilon_t = \int_{L_0}^{L_i} \frac{dl}{l} = \ln \frac{L_i}{L_0}$$



As:

$$\epsilon_n = \frac{L_i - L_0}{L_0} = \frac{L_i}{L_0} - 1 \Rightarrow \frac{L_i}{L_0} = 1 + \epsilon_n$$

$$\boxed{\epsilon_t = \ln(1 + \epsilon_n)}$$

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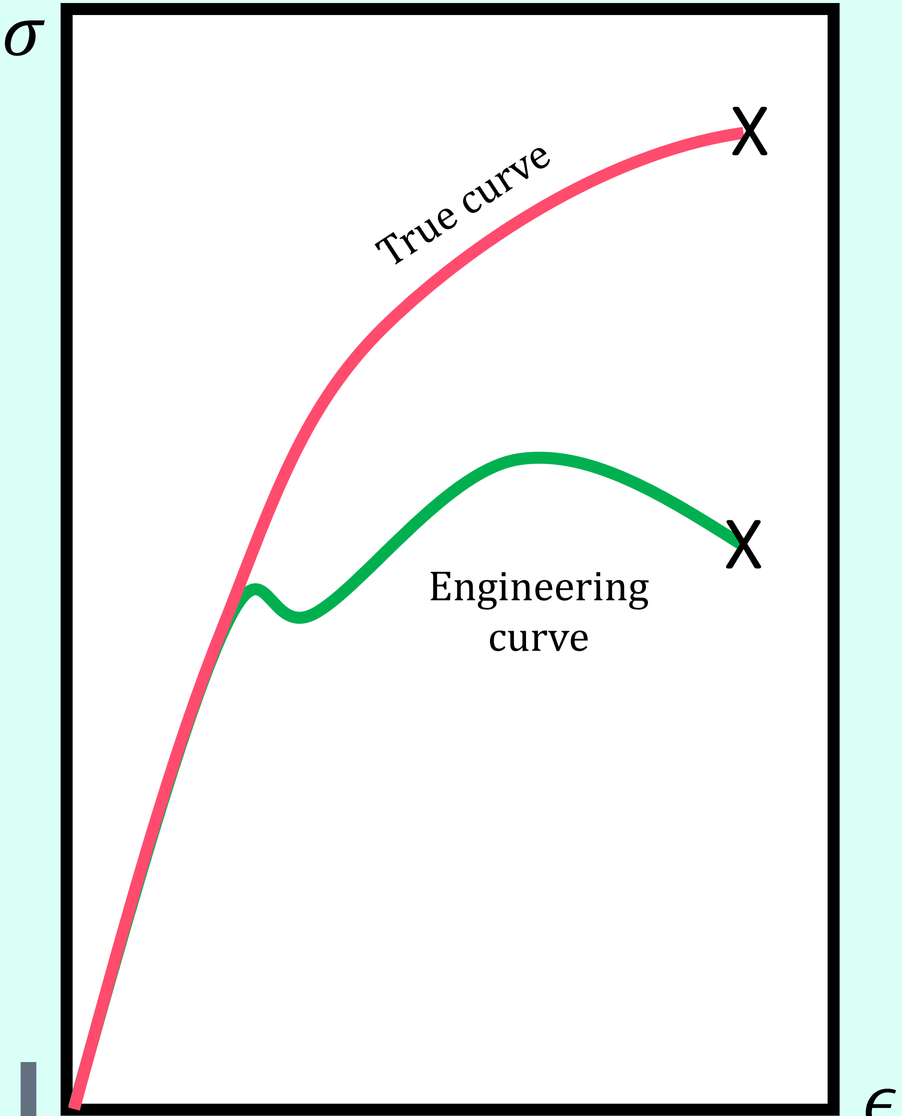
Comparison

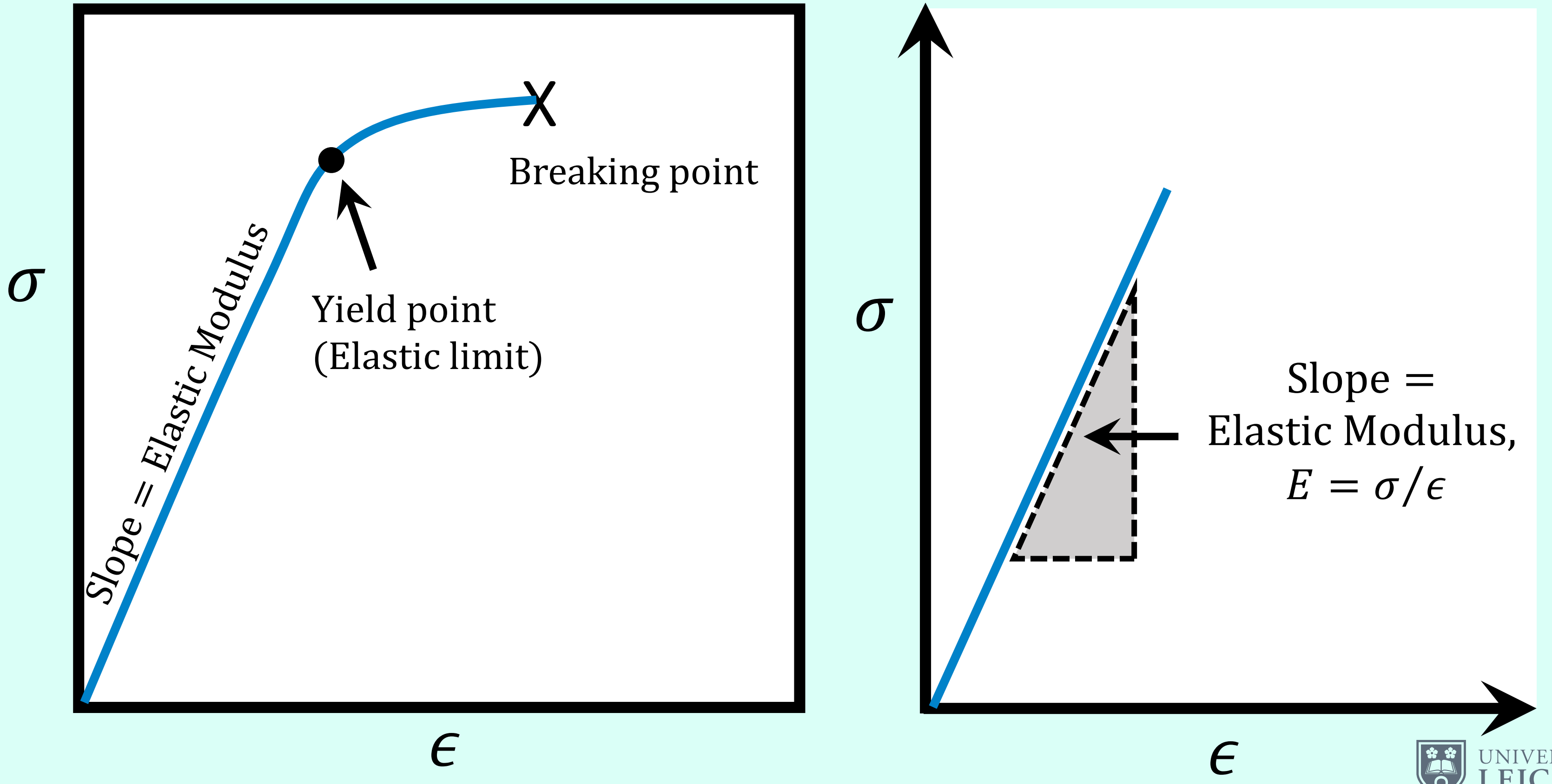
$\epsilon_t = \ln(1 + \epsilon_n)$ $\sigma_t = \sigma_n(1 + \epsilon_n)$



- True curve:
  - Continuous increase in stress and strain
  - Instantaneous values considered – actual stresses in the material are calculated
- Engineering curve:
  - Drop in stress due to rapid onset of necking
  - Larger scale necking follows

Relationship between Stress and Strain – Young's Modulus

The **<u>Elastic (Young's) Modulus, E</u>** – the stiffness or the resistance of a material to elastic deformation.

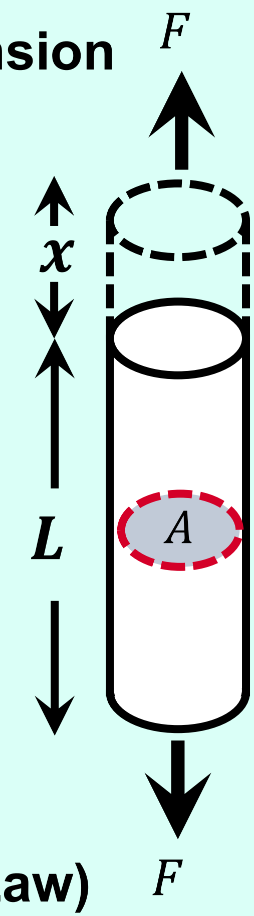


Relationship between Stress and Strain – Hooke's Law

Hooke's Law: 'Ut tensio sic vis' (as the extension, so the force)
The basic statement of linear elasticity formulated by Robert Hooke in the 17th century – **linear relationship between force and extension**

$$F \propto x \Rightarrow F = kx$$

Where k is the force constant (Stiffness).



In order to derive a materials property:

- Divide through by Area (A) to get **Stress (σ)**:

$$\sigma = \frac{kx}{A}$$

- Multiply and divide by Length (L) to get **Strain (ϵ):**

$$\sigma = \frac{kL}{A}\epsilon \Rightarrow \frac{kL}{A} = \frac{\sigma}{\epsilon}$$

Young's (or Elastic) Modulus E (1807): $E = \frac{\sigma}{\epsilon}$ **(Hooke's Law)**

<u>**This is defined as Linear Elastic Behaviour**</u>

Typical Values of the Elastic Modulus

- The elastic modulus of engineering materials
- Modulus limited design case studies

Material	E ($GN\,m^{-2}$)	Material	E ($GN\,m^{-2}$)
Diamond	1000	Niobium and alloys	80–110
Tungsten carbide, WC	450–650	Silicon	107
Osmium	551	Zirconium and alloys	96
Cobalt/tungsten carbide cermets	400–530	Silica glass, SiO_2 (quartz)	94
Borides of Ti, Zr, Hf	450–500	Zinc and alloys	43–96
Silicon carbide, SiC	430–445	Gold	82
Boron	441	Calcite (marble, limestone)	70–82
Tungsten and alloys	380–411	Aluminium	69
Alumina, Al_2O_3	385–392	Aluminium and alloys	69–79
Beryllia, BeO	375–385	Silver	76
Titanium carbide, TiC	370–380	Soda glass	69
Tantalum carbide, TaC	360–375	Alkali halides (NaCl, LiF, etc.)	15–68
Molybdenum and alloys	320–365	Granite (Westerly granite)	62
Niobium carbide, NbC	320–340	Tin and alloys	41–53
Silicon nitride, Si_3N_4	280–310	Concrete, cement	30–50
Beryllium and alloys	290–318	Fibreglass (glass-fibre/epoxy)	35–45
Chromium	285–290	Magnesium and alloys	41–45
Magnesia, MgO	240–275	GFRP	7–45
Cobalt and alloys	200–248	Calcite (marble, limestone)	31
Zirconia, ZrO_2	160–241	Graphite	27
Nickel	214	Shale (oil shale)	18
Nickel alloys	130–234	Common woods, ∥ to grain	9–16
CFRP	70–200	Lead and alloys	16–18
Iron	196	Alkyds	14–17
Iron-based super-alloys	193–214	Ice, H_2O	9.1
Ferritic steels, low-alloy steels	196–207	Melamines	6–7
Stainless austenitic steels	190–200	Polyimides	3–5
Mild steel	200	Polyesters	1.8–3.5
Cast irons	170–190	Acrylics	1.6–3.4
Tantalum and alloys	150–186	Nylon	2–4
Platinum	172	PMMA	3.4
Uranium	172	Polystyrene	3–3.4
Boron/epoxy composites	80–160	Epoxies	2.6–3
Copper	124	Polycarbonate	2.6
Copper alloys	120–150	Common woods, ⊥ to grain	0.6–1.0
Mullite	145	Polypropylene	0.9
Vanadium	130	PVC	0.2–0.8
Titanium	116	Polyethylene, high density	0.7
Titanium alloys	80–130	Foamed polyurethane	0.01–0.06
Palladium	124	Polyethylene, low density	0.2
Brasses and bronzes	103–124	Rubbers	0.01–0.1
		Foamed polymers	0.001–0.01

M.F. Ashby and D.R.H. Jones, Engineering Materials 1, Butterworth Heinemann 1996

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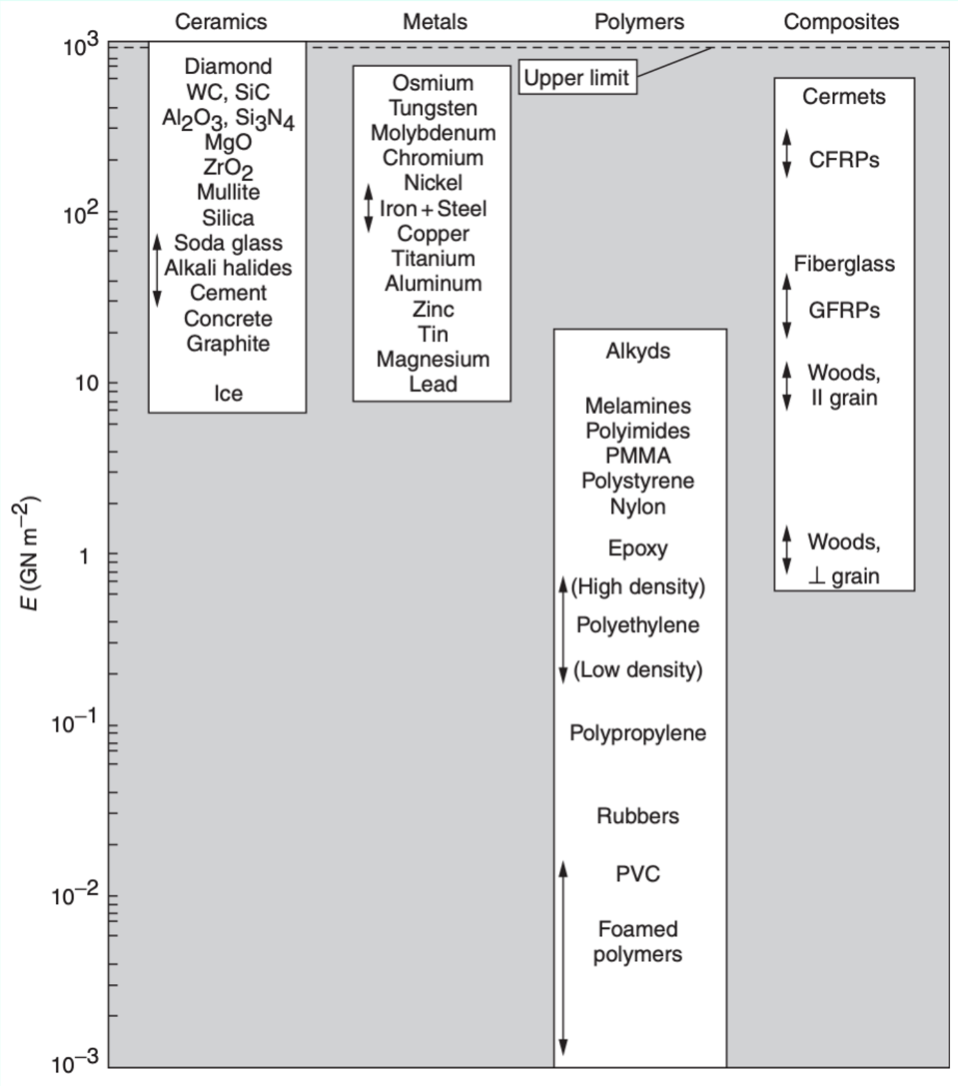
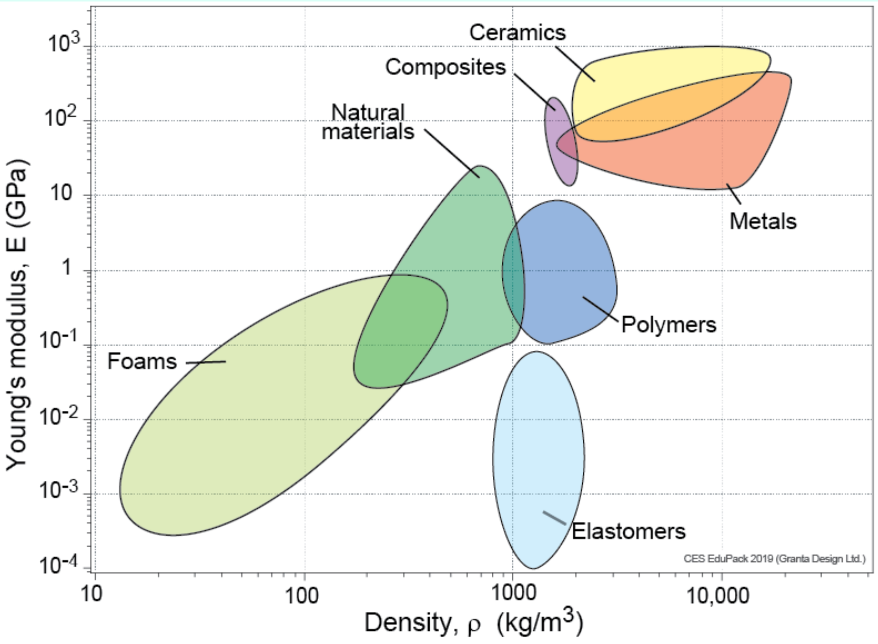
Young’s modulus



Young's Modulus vs Density Chart. Material Family Chart



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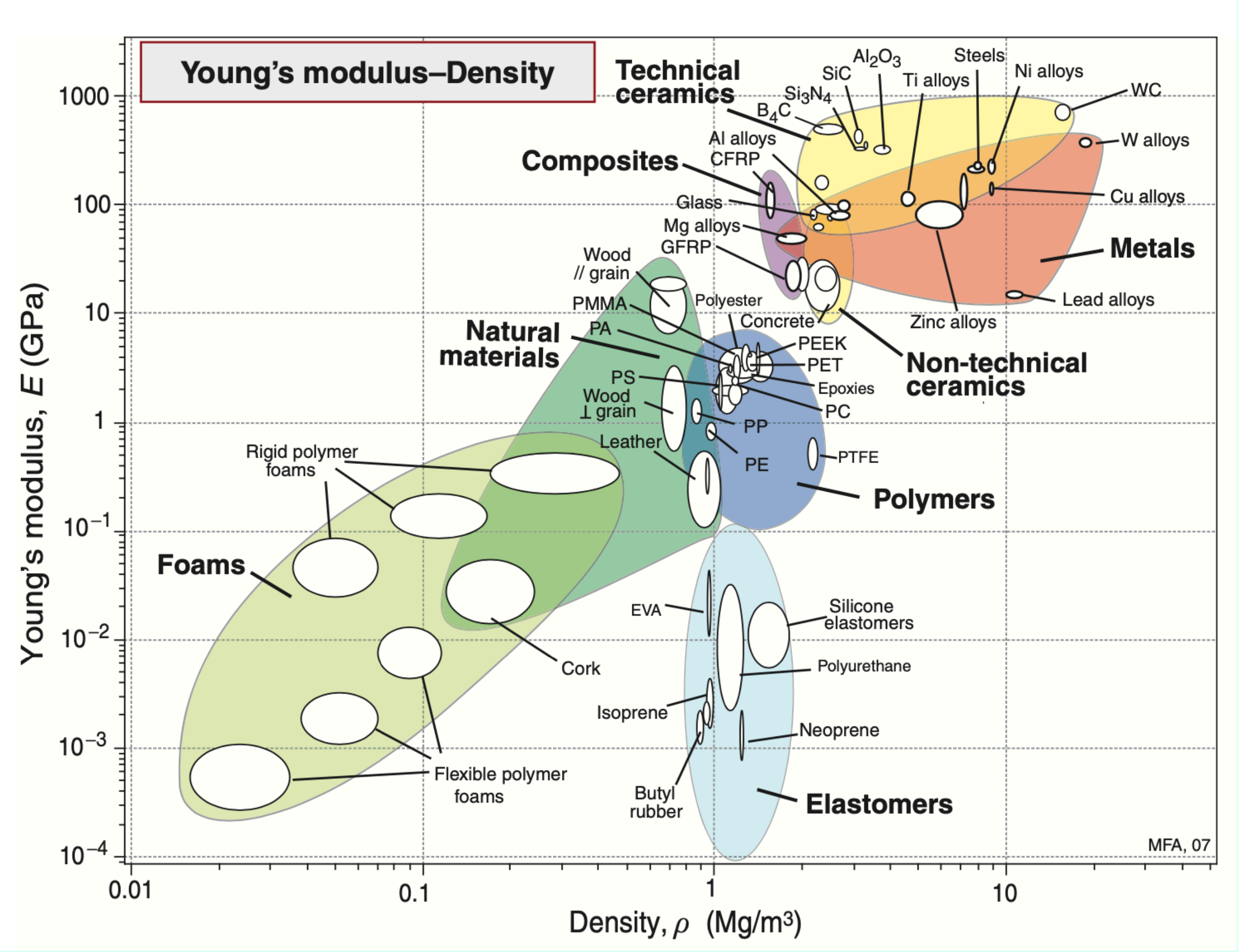
Typical Values of the Elastic Modulus



Logarithmic Axes

Example 1

Steel Control Cable in Aircraft has Diameter of $3\ \text{mm}$.
Load on Cable is 1kN. Modulus of Elasticity for Steel is: $200\ \text{GPa}$

$$\sigma = \frac{Force}{Area} = \frac{1000}{\pi\left(\frac{3}{2}\right)^2} = 141.47\ MPa$$

$$\sigma = E.\epsilon$$

$$\epsilon = \frac{\sigma}{E} = \frac{141.47}{200000} = 0.00070735$$

Strains often expressed as a percentage (%) – giving $\epsilon = 0.071\%$

Note: Stress in MPa = N/mm^2

Example 2

Displacement (Extension) at the end of the 3 mm diameter cable under 141.47 MPa tensile stress from 1kN load
Giving a calculated strain of 0.071% (0.00071 real value)

$$\epsilon = \Delta L / L$$

If the steel Cable is 10 m long then extension of cable ΔL is:

$$\Delta L = L * \epsilon = 0.00071 * 10000 = 7.07\, mm$$

Note: conversion of cable length from meters to millimeters and strain from % to real value

Example 2

The lateral strain on the cable is given by: $\epsilon_x = -\nu * \epsilon_y$

Assume the material has a Poisson's ratio (ν) of 0.3

$$\epsilon_x = -\nu\epsilon_y = -0.3 * 0.00070735 = -0.00021221$$

The diameter of the cable under this lateral strain (d') is:

$$d' = d - \Delta d = d - (\epsilon_x * d) = 3 - (0.00021221 * 3) = 2.999363 \text{ mm}$$

This corresponds to a reduction in cross-section area of the cable by a factor of 0.999575

This gives a very small increase in stress in the cable to 141.53 MPa