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School of Engineering

Pin Jointed Frames

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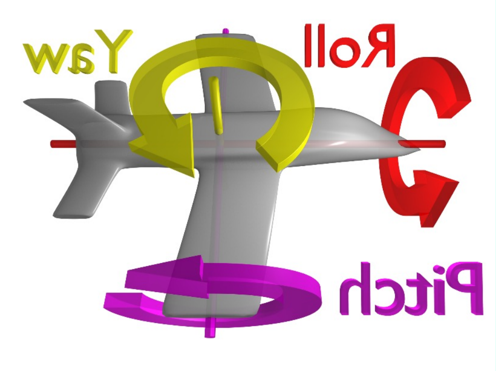
Rigid Body



<u>Rigid Body</u> defined as:

- Solid Body whose Deformation is either Zero or Negligible
  i.e. Deformation so small that it can be ignored

- Distance between any 2 Points in Body effectively Constant
  Regardless of any External Forces

- **Rigid Body** considered as Continuous Distribution of Mass

Statics

- Concerned with Analysis of Loads (Force and Torque, or ‘Moment’)
- Forces assumed to be in equilibrium (balance) within a body
- Body does NOT experience an Acceleration ($\underline{a} = \underline{0}$)
- Condition known as ‘**Static Equilibrium**’
- System is ‘**at rest**’ or ‘**moving at a constant velocity**’

e.g. Stationary Objects

Buildings, Bridges etc.

Objects in Stable Motion (constant velocity)

Aircraft in stable flight, Car cruising on motorway etc.

Static Equilibrium

Thus, for '**Static Equilibrium**' Conditions

No Linear Acceleration of the Body

$$\sum_i \underline{\boldsymbol{F}_i} = \underline{\mathbf{0}}$$

No Angular Acceleration of the Body

$$\sum_i \underline{\boldsymbol{M}_i} = \underline{\mathbf{0}}$$

Moment of a Force

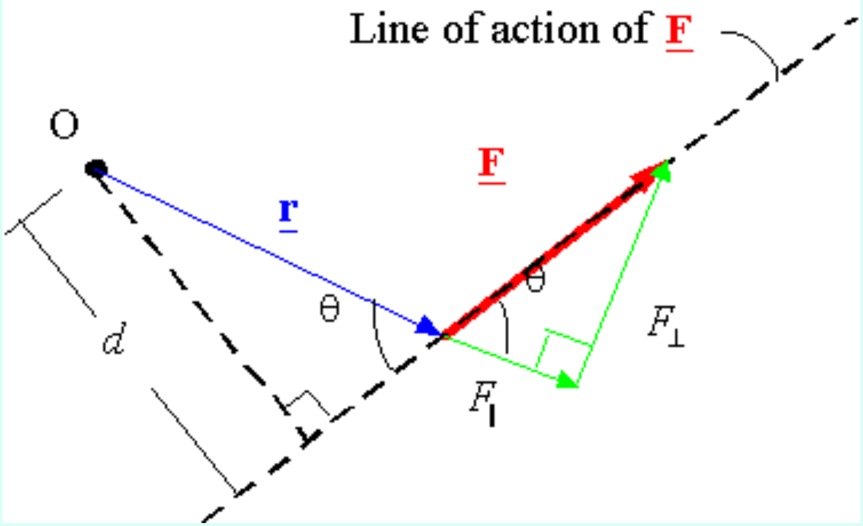
Force can also **ROTATE** a body about an AXIS or Point

Rotational Tendency known as: ***Moment*** (***M***) of the Force

(Moment can also be referred to as ***Torque***)

Moment of a Force

Moment of a Force about a Point O



Magnitude of the Moment of Force (M) about Point O given by:

$$M_O = F.d$$

where

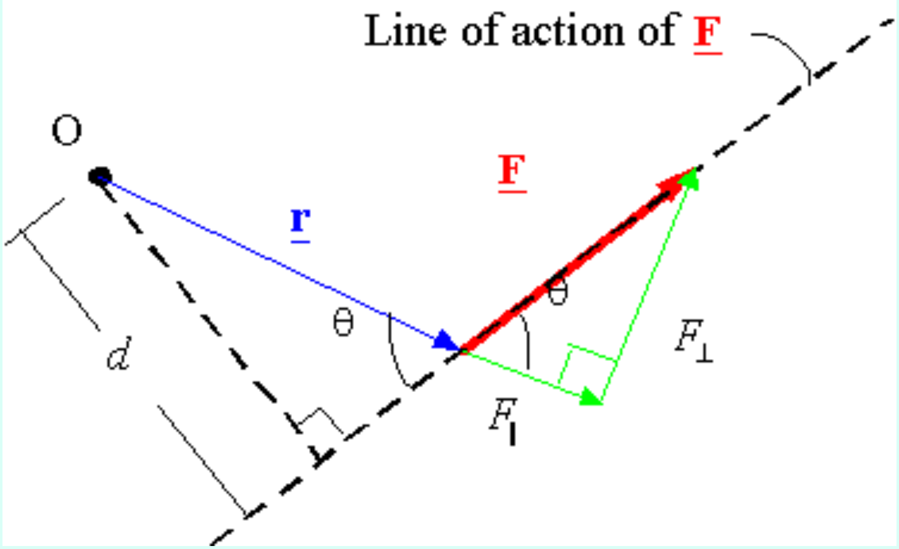
F is the Magnitude of Applied Force

d is **perpendicular** distance from the line of action of the Force

Note: Sign Convention for direction of Moments must be consistent in a given calculation

Moment of a Force

Moment of a Force about a Point O



In Vector Format, Moment ($\underline{M}$) given by the **Vector Cross Product**:

$$\underline{M_O} = \underline{r} \times \underline{F}$$

where

$\underline{F}$ is the Force Vector

$\underline{r}$ is the radius vector from the Point O to the line of action

Free Body Diagrams

- **Shows the Forces and Moments on a Body**
- Enables Calculation of the Resulting Reaction Forces
- Used to Determine the Loading of Individual Structural Components
- Also Calculates Internal Forces within a Structure
- Essentially a **VECTOR** diagram of all localized Forces
- Condition of **Static Equilibrium** assumed
    - **i.e. Sum of Forces and Moments must be zero**

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Free Body Diagrams

- Simplified Version of Structural Component
    - Often a Point, Line or Box
- Forces shown as Arrows pointing in direction they act on Body
- Moments shown as Curved Arrows in direction they act on Body
- Coordinate System
- Reactions to Applied Forces also Shown

Free Body Diagrams

- Typically Provisional Free Body Diagram drawn before all Forces and Reactions are known so that unknowns can be evaluated
- Constraints replaced by Reaction Forces
- Note: If External Forces are small → Can Be Neglected
    - Buoyancy forces in Air
    - Atmospheric Pressure
- Free Body analysed by Summing all the Forces
    - Resolved into the coordinate system directions
    - Net Force in any direction is Zero for Static Equilibrium: $\sum \boldsymbol{F_x} = \mathbf{0} \quad \sum \boldsymbol{F_y} = \mathbf{0}$
    - Net Moment is Zero for Static Equilibrium: $\sum \boldsymbol{M} = \mathbf{0}$

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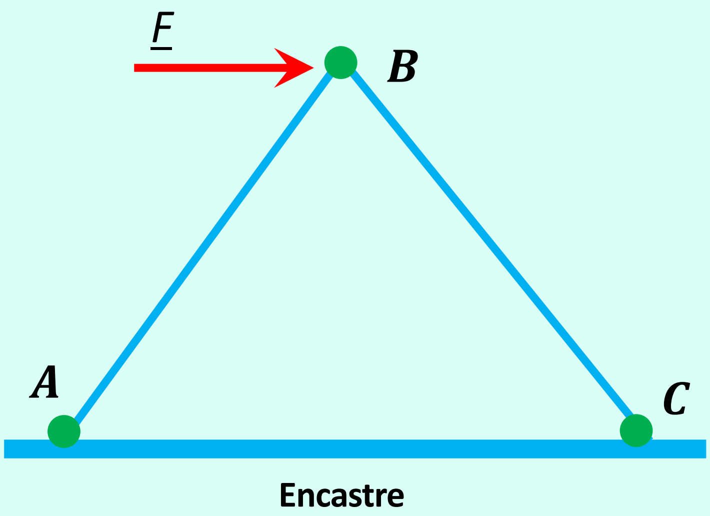
Free Body Diagrams

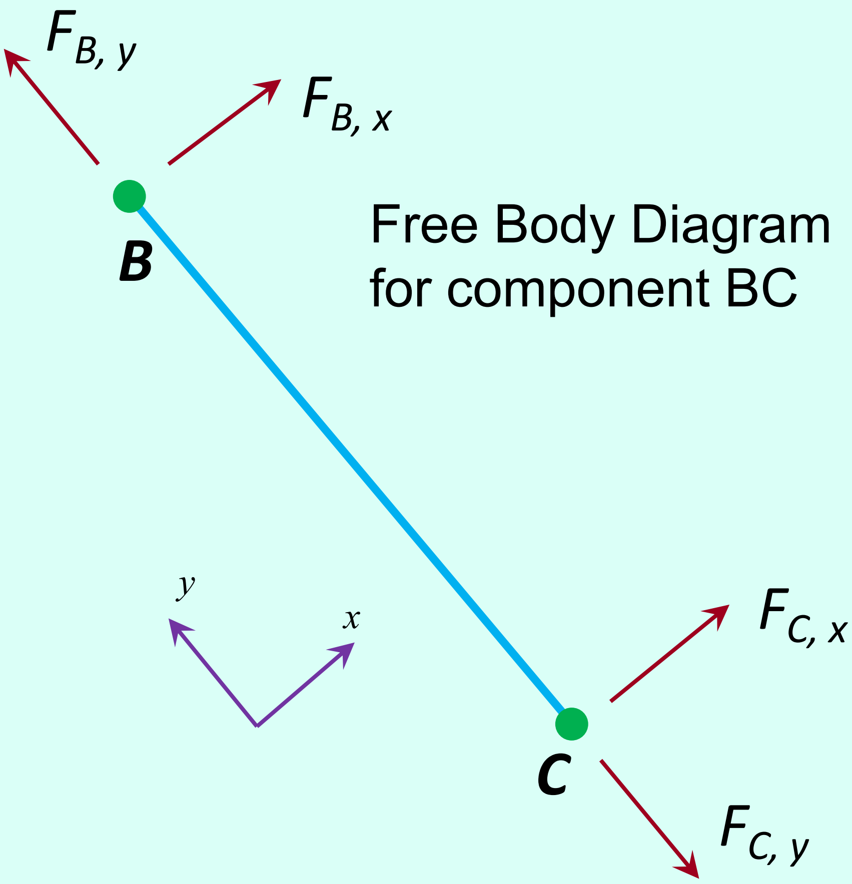
A free body diagram consists of:

- A coordinate system
- A simplified version of the isolated body
- Forces shown as straight arrows pointing in the direction they act on the body
- Moments shown as curved arrows pointing in the direction they act on the body
- Supports are replaced by reaction forces and moments

Free body diagrams can easily be constructed for simple problems

Free Body Diagrams: Simple Example





Free Body Diagram for component BC

Free Body Diagrams: Simple Example

Balance of Forces

Along Axis of Bar ***BC***

$$F_{B,y} - F_{C,y} = 0$$

Note: $F_{C,\,y}$ is a force in the negative y-direction

Balance of Moments

Taking Moment about Point B

Length of Bar BC is l_{BC}

$$0 + 0 + 0 + F_{C,x}.\, l_{BC} = 0$$

Which Implies $F_{C,x} = 0$

Similarly $F_{B,x} = 0$

if we take Moment about Point C.

Conclusion: a solid bar (member) in a pin-jointed structure does not carry any forces <u>perpendicular</u> to the axis of the bar

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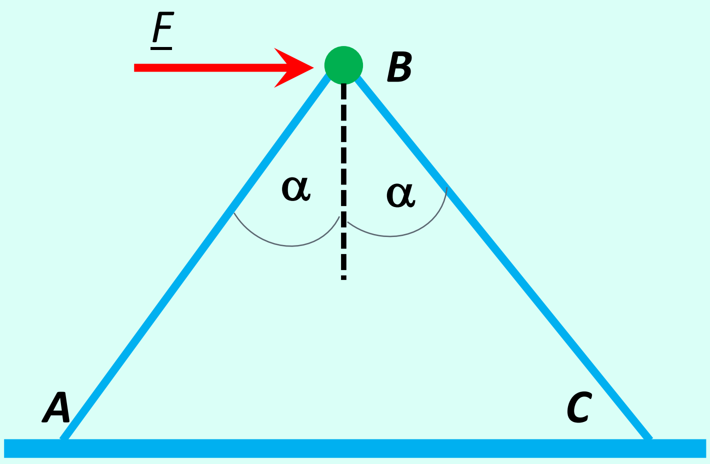
Pin Jointed Structures

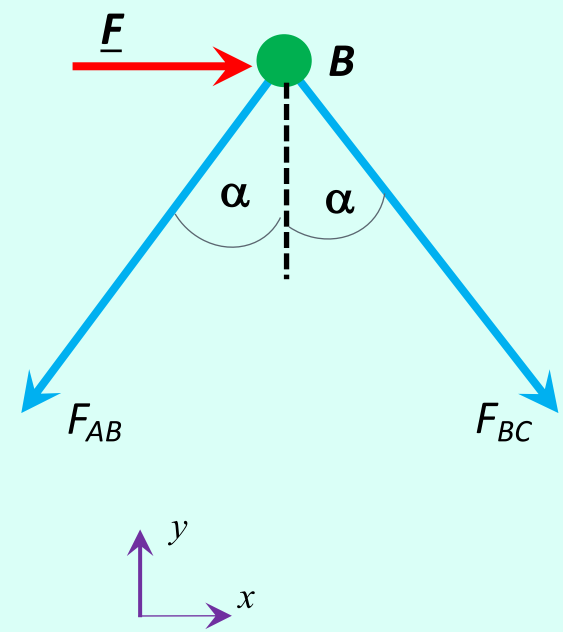
Free to Rotate at the Joints between Structural Members

- Solid Bar (member) in a Pin-Jointed Structure does not carry any Forces perpendicular to the axis of the bar

Pin Jointed Structures: Simple Example



Taking Joint B as a Free Body Diagram



Pin Jointed Structures: Simple Example

At Point B

Balance of Forces in x-direction

$$F + F_{BC}\sin\alpha - F_{AB}\sin\alpha = 0$$

Balance of Forces in y-direction

$$-F_{BC}\cos\alpha - F_{AB}\cos\alpha = 0$$

which gives: $F_{AB} = -F_{BC}$

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Pin Jointed Structures: Simple Example

Then, By Substitution

$$F + F_{BC}\sin\alpha + F_{BC}\sin\alpha = 0$$

giving

$$F_{BC} = -\frac{F}{2.\sin\alpha}$$

thus

$$F_{AB} = -F_{BC} = \frac{F}{2.\sin\alpha}$$